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COAP 2022 Best Paper Prize

Each year, the editorial board of Computational Optimization and Applications selects a paper from the preceding year's publications for the Best Paper Award. In 2022, 84 papers were published in the journal. This article highlights the research related to the award winning work of Alberto De Marchi (University of the Bundeswehr Munich) in his paper "On a primal-dual Newton proximal method for convex quadratic programs", published in volume 81, pages 369–395.

De Marchi's paper [6] describes a numerical method for convex quadratic programs (QPs), that is, optimization problems with quadratic objective function and polyhedral constraints. The proposed iterative scheme, called `QPDO`, combines the proximal point algorithm and a globalized Newton-type method, tailored to each other. Both the inner problems and solver are formulated and designed to let the regularization properties and linear structures shine. These interactions yield a numerically stable and robust method and allows to exploit efficient linear algebra routines.

Although seemingly simple, QP covers a wide spectrum of applications and, despite convexity, comes with computational challenges when the problem data fail to satisfy additional properties. Setting aside the possibility (of detecting) primal and/or dual infeasibility, numerical difficulties originate from the lack of positive definiteness or linear independence of the constraints. Seeking a robust yet efficient method, the general-purpose approach presented in [6] does not demand such stipulations to reliably solve QPs. Proximal regularization schemes have become increasingly popular in QP solvers to deal with these issues while handling both large scale and embedded optimization problems [2, 15, 18]. De Marchi's work closely investigates the interactions between (outer) proximal regularization and (inner) linear systems, seeking to design numerical methods that benefits from them to gain robustness and scalability.

With [6] this vision translates into the careful interlocking of the well-known proximal point algorithm (PPA) and Newton methods. Necessary optimality conditions are also sufficient for convex QP, thus playing a fundamental role for solvers. Such KKT conditions can be written as a monotone inclusion and as a system of piecewise affine equations, making respectively proximal and Newton methods attractive. Building upon the inexact PPA [16] as in `FBstab` [15], `QPDO` deviates from the latter in reformulating the complementarity conditions using the minimum instead of the Fischer-Burmeister function. This unusual choice has several advantages, above all it avoids to introduce nonlinearities and leads to symmetric linear systems, but Newton methods for piecewise affine equations are more difficult to globalize. The observation for solving this puzzle was as much simple as crucial: unveiling that the primal-dual augmented Lagrangian function [12] can be a suitable merit function for globalization, De Marchi's contribution enables the adoption of efficient linear algebra routines for solving numerically stabilized linear systems [19] within a globalized semismooth Newton method. The goal of the numerical experiments in [6] was to validate and compare the robustness and performance of this algorithmic approach for QP. Despite the relatively simple scheme and implementation, `QPDO` delivered encouraging results, analogous to `OSQP` [18] and `QPALM` [13], and was particularly competitive on difficult instances.

There exist several related works and, since the publication of [6] in Computational Optimization and Applications, the proposed approach has already served as a foundation or inspiration for the development of methods for more general problem classes. Prominent examples are the closely related `PROX-QP` [1] and the extension `PROX-NLP` [14] for nonlinear programming, on the vein of De Marchi's own papers [5, 8]. It is worth mentioning also the influence of [6] on the evolution from `IP-PMM` [17] to `PS-IPM` [3], where the recombination of proximal and interior point schemes leads not only to improved efficiency and robustness, but also benefits the preconditioning of iterative linear solvers.

Finally, the scope of De Marchi's algorithmic design approach goes well beyond quadratic and nonlinear programming. As discussed in his doctoral dissertation [4], the results in [6] stem from bringing together augmented Lagrangian and proximal frameworks for addressing more general problems and then specializing to QP and exploiting their structure. Extensions to fully nonconvex composite optimization have been only recently studied [7, 9–11] and remain a challenging topic for further investigation.

Alberto De Marchi is currently a postdoctoral researcher at the Institute for Applied Mathematics and Scientific Computing at the University of the Bundeswehr Munich, Germany, where he received his doctoral degree (Dr. rer. nat.) in June 2021. He was a visiting researcher at Curtin University, Australia (Fall 2022) and Kyushu University, Japan (Summer 2023). His scientific activity revolves around computational optimization, with applications ranging from control systems to data science.

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