Comput Optim Appl (2016) 65:541–544
DOI 10.1007/s10589-016-9882-5

COAP 2015 Best Paper Prize

Published online: 1 November 2016

Each year, the editorial board of computational optimization and applications (COAP) selects a paper from the preceding year's publications for the Best Paper Award. In 2015, 92 papers were published by COAP. The recipients of the 2015 Best Paper Award are Qi Huangfu of the FICO Xpress Team and Julian Hall of the University of Edinburgh for their paper "Novel update techniques for the revised simplex method" published in volume 60, pages 587–608. We point out that Julian Hall has now won 3 of the 14 Best Paper Awards given out since the inception of the award in 2003, a remarkable achievement. This article highlights the research related to the award winning paper.

The paper introduces three novel techniques for updating the invertible representation of the basis matrix when solving practical sparse linear programming (LP) problems using a high performance implementation of the dual revised simplex method. These are of particular value when suboptimization is used. Two are variants of the product form update and the other permits multiple Forrest-Tomlin updates to be performed.

The development of an efficient parallel implementation of the revised simplex method has attracted the attention of leading researchers over the past 30 years, but with relatively little success [4] until recently. For example, Hall and McKinnon devised two parallel schemes for the primal revised simplex method in the mid 1990s [6,7]. Their moderately efficient prototype implementation gave limited speedup on general sparse LP problems, but the performance was well below that of the best serial solvers of the time. A byproduct of this work was the identification of hyper-sparsity in the simplex method and techniques for its exploitation using serial computing [8], work which received the COAP Best Paper Award for 2005. A few years ago, Lubin, Hall, Petra and Anitecu developed a parallelization of the revised simplex method for large scale two-stage stochastic LP problems [10]. The performance of their distributed memory application scaled linearly on hundreds of nodes and enabled LP problems with hundreds of millions of variables to be solved and they received the COAP Best Paper Award for 2013. However, for general large scale sparse LP problems, the search continued for techniques which would yield an efficient parallel implementation of the revised simplex method.

When Huangfu began his PhD under the supervision of Hall in 2009, his aim was to explore the scope for parallelism offered by suboptimization [12], which uses minor iterations of the standard dual simplex method within the revised dual simplex method. This requires the solution of independent linear systems before and after the minor iterations, a source of task parallelism even when each particular system is solved serially. Using modules of his own serial dual revised simplex solver based on relatively inefficient and numerically unstable product form updates, he developed a prototype parallel implementation of the dual revised simplex method with suboptimization [5]. This achieved a mean speedup of 30 % on four or eight cores with the eight-core performance being about a factor of two below that of `Clp` [1].

For the parallel solver to be competitive with good serial solvers it was necessary to base it on the Forrest-Tomlin update [2] of the invertible representation of the basis matrix. The need to perform multiple Forrest-Tomlin updates efficiently after the minor iterations drove the development of one of the techniques described by Huangfu and Hall in the Prize Paper. Following the minor iterations it is also necessary to update the primal variables using a linear combination of tableau columns, each corresponding to a different basis. Where the traditional product form update takes the basis matrix B as a factor on the left of the expression $\bar{B} = B + (\boldsymbol{a}_q - B\boldsymbol{e}_p)\boldsymbol{e}_p^T$ for the updated basis matrix, the "alternative" product form update introduced in the Prize Paper takes B as a factor on the *right*. This allows the primal variables to be updated with essentially a single system solve. These two techniques were crucial to the efficiency of the ultimate parallel dual revised simplex solver, `hsol`, developed by Huangfu [9]. This solver out-performed `Clp` by a factor of 1.6 in serial and 2.4 using eight cores on a set of 30 significant representative test problems.

Upon completion of his PhD in 2013, Huangfu moved to the FICO Xpress Team where his successful implementation of some of the techniques developed in Edinburgh within the Xpress dual simplex solver has featured in FICO promotional material and Xpress now sits at the top of Mittelmann's simplex benchmarks [11]. The final novel update in the Prize Paper is the second natural variant of the product form update. Although it is not used in `hsol`, Google's LP solver `Glop` [3] employs it to update the invertible representation of the basis matrix. As for `hsol`, Hall and Galabova are adding features such as presolve, crash and advanced basis start to make it more widely useful. Once this work is complete, `hsol`will be released via COIN-OR as a high performance open-source simplex solver. Not only is it expected to out-perform `Clp` but, by being written in well-structured and documented C++, it will offer a true open-source platform for application and further experimentation.

This award is dedicated to the memory of Roger Fletcher, whose influence on computational optimization is immense and the discovery of whose book, "Practical Methods of Optimization" set Julian Hall on the road to a PhD under his supervision in Dundee and a career in optimization.

References

1. COIN-OR. Clp. http://www.coin-or.org/projects/Clp.xml (2014). Accessed 11 Oct 2016
2. Forrest, J.J.H., Tomlin, J.A.: Updated triangular factors of the basis to maintain sparsity in the product form simplex method. Math. Progr. **2**, 263–278 (1972)
3. Google. Glop. https://developers.google.com/optimization/lp/glop (2016). Accessed 11 Oct 2016
4. Hall, J.A.J.: Towards a practical parallelisation of the simplex method. Comput. Manag. Sci. **7**(2), 139–170 (2010)
5. Hall, J.A.J., Huangfu, Q.: A high performance dual revised simplex solver. In: R. W., et al. (eds.) PPAM 2011. Part I, vol. 7203 of LNCS, pp. 143–151. Springer, Heidelberg (2012)
6. Hall, J. A. J., McKinnon, K. I. M.: PARSMI, a parallel revised simplex algorithm incorporating minor iterations and Devex pricing. In: Waśniewski, J., Dongarra, J., Madsen, K., Olesen, D. (eds.) , Applied parallel computing, vol. 1184 of lecture notes in computer science, pp. 67–76. Springer, Berlin (1996)
7. Hall, J.A.J., McKinnon, K.I.M.: ASYNPLEX, an asynchronous parallel revised simplex method algorithm. Ann. Oper. Res. **81**, 27–49 (1998)
8. Hall, J.A.J., McKinnon, K.I.M.: Hyper-sparsity in the revised simplex method and how to exploit it. Comput. Optim. Appl. **32**(3), 259–283 (2005)
9. Huangfu, Q., Hall, J. A. J.: Parallelizing the dual revised simplex method. Mathematical programming computation, Accepted for publication (2016)
10. Lubin, M., Hall, J.A.J., Petra, C.G., Anitescu, M.: Parallel distributed-memory simplex for large-scale stochastic LP problems. Comput. Optim. Appl. **55**(3), 571–596 (2013)
11. Mittelmann, H. D.: Benchmark of simplex lp solvers. http://plato.asu.edu/ftp/lpsimp.html(2016). Accessed 11 Oct 2016
12. Rosander, R.R.: Multiple pricing and suboptimization in dual linear programming algorithms. Math. Progr. Stud. **4**, 108–117 (1975)

Qi Huangfu received his B.Sc. in Computer Science and Technology from Beihang University in 2007 and an M.Sc. in High Performance Computing from the University of Edinburgh in 2008, before graduating with a Ph.D. in Mathematics and Statistics from the University of Edinburgh in 2013. He is currently a Senior Engineer at FICO in Birmingham, UK.

Julian Hall received his B.A. in Mathematics from the University of Oxford in 1987 and his Ph.D. from the University of Dundee in 1992. Since 1990 he has been employed as a lecturer in the School of Mathematics at the University of Edinburgh.