Comput Optim Appl (2009) 44: 521–524
DOI 10.1007/s10589-009-9304-z

COAP 2008 best paper award: Paper of M. Weiser, T. Gänzler, and A. Schiela

W.W. Hager

Received: 3 November 2009 / Published online: 19 November 2009

Since 2004, the Computational Optimization and Applications (COAP) editorial board has selected a paper from the preceding yearâs COAP publications for the 'Best Paper Award.' The award competition among papers published in 2008 culminated in a tie between two papers. This article concerns the award winning work of Martin Weiser, Tobias Gänzler, and Anton Schiela at the Zuse Institute Berlin and their paper [5] "A control reduced primal interior point method for a class of control constrained optimal control problems", published in Volume 41, Issue 1, pages 127–145.

This paper as well as its sequel [3] presents work done at the Zuse Institute and DFG Research Center MATHEON while Tobias and Anton were PhD students working on the optimization of hyperthermia treatment planning. Regional hyperthermia is a cancer therapy aiming at heating the tumor by absorption of electromagnetic fields to make it more susceptible to an accompanying radio or chemo therapy. Since the physical and physiological processes are quite complex, individual therapy planning requires numerical optimization [1, 4].

The article deals with interior point methods applied to control constrained PDE optimal control problems. The prototype problem is

$$\begin{aligned} \min \quad & \|y - y_d\|^2 + \alpha \|u\|^2 \\ \text{s.t.} \quad & -\Delta y = u, \quad -1 \le u \le 1. \end{aligned}$$

The aim was to establish function space oriented algorithms which work directly in the infinite-dimensional setting, using adaptively refined finite element discretizations as a tool to compute inexact Newton corrections. While the linear convergence

W.W. Hager (✉)
Department of Mathematics, University of Florida, Gainesville, FL 32611-8105, USA
e-mail: hager@ufl.edu

of such short-step methods had already been established, the computational performance was impeded by massive mesh refinements along the boundary of the active set. This refinement is necessary, at least for small regularization parameter α, due to the large curvature of u which appears for the interior point barrier parameter μ going towards zero.

At this time, Michael Hinze proposed to exploit the well-known projection formula

$$u = \max(-1, \min(\lambda/\alpha, 1))$$

connecting the control with the adjoint variable λ in the context of active set methods [2]. This non-smooth elimination of the control leaves the state y and the adjoint variable λ as independent variables in the optimality system. In particular, the low regularity of the control no longer dominates the mesh refinement.

The authors transfered this idea to interior point methods, where the usual log-barrier transforms the projection formula into the cubic equation

$$\alpha u - \frac{u}{u+1} + \frac{\mu}{1-u} = \lambda.$$

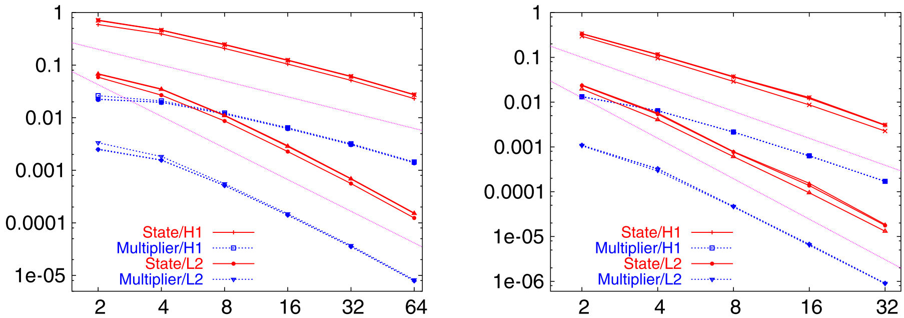

Fig. 1 Convergence of the control reduced interior point method on a test problem uniformly discretized with linear (*left*) and quadratic (*right*) finite elements. H^1 and L^2 errors of the remaining state and adjoint variables for values of $\mu = 2^{-8}, 2^{-13}, 2^{-15}$

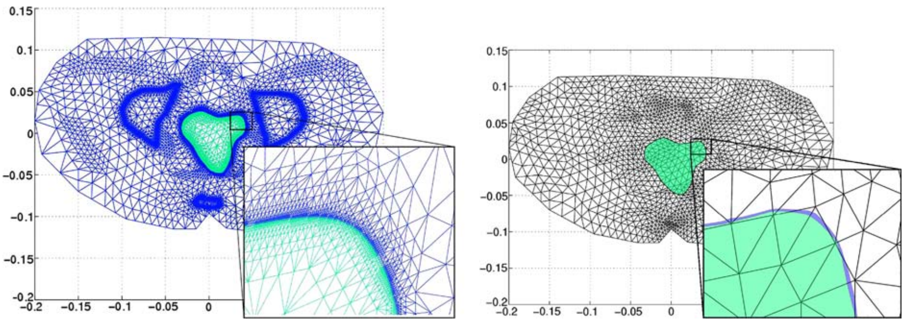

Fig. 2 Adaptively refined meshes and boundary of the active set for a simplified hyperthermia model problem

Due to monotonicity, a unique solution exists and can be used to eliminate the control as in the active set method. This elimination, dubbed control reduction in analogy to reduced methods where the state is treated as dependent variable, turned out to be the key to efficiency of function space oriented interior point methods. In the article it has been shown that with control reduction and a proper equilibration of discretization and barrier error, the convergence rate of linear and quadratic finite elements for state and adjoint variables are not affected by the large control curvature at the boundary of the active set (see Fig. 1). Thus, the same accuracy can be obtained on much coarser meshes (see Fig. 2). Moreover, control reduction allows to achieve superlinear convergence of function space oriented interior point methods [3].

References

1. Deuflhard, P., Seebaß, M., Stalling, D., Beck, R., Hege, H.-C.: Hyperthermia treatment planning in clinical cancer therapy: modelling, simulation and visualization. In: Sydow, A. (ed.) Proc. of the 15th IMACS World Congress 1997 on Scientific Computation: Modelling and Applied Mathematics, vol. 3, pp. 9–17. Wissenschaft und Technik Verlag (1997)
2. Hinze, M.: A variational discretization concept in control constrained optimization: the linear-quadratic case. Comput. Optim. Appl. **30**, 45–63 (2005)
3. Schiela, A., Weiser, M.: Superlinear convergence of the control reduced interior point method for PDE constrained optimization. Comput. Optim. Appl. **39**(3), 369–393 (2008)
4. Weiser, M.: Optimization and identification in regional hyperthermia. Int. J. Appl. Electromagn. Mech. **30**, 265–275 (2009)
5. Weiser, M., Gänzler, T., Schiela, A.: A control reduced primal interior point method for a class of control constrained optimal control problems. Comput. Optim. Appl. **41**(1), 127–145 (2008)

Martin Weiser received his Diploma in Mathematics in 1996 and his PhD in 2001 from Freie Universität Berlin. Since 1996 he is researcher at the Zuse Institute Berlin, where he currently heads the Computational Medicine group. His main research interests are optimal control with PDEs and biomedical applications.

Tobias Gänzler received his degree in Mathematics in 1999 from the University of Würzburg, Germany. From 2002 to 2006 he worked as a scientific assistant at the Zuse Institute Berlin. During that time he was a member of the DFG Research Center MATHEON "Mathematics for key technologies". Since 2006 he is employed as a software developer in Hamburg.

Anton Schiela received his Diploma in Mathematics in 2002 from TU München and his PhD in 2007 from Freie Universität Berlin. Since 2002 he is researcher in the Computational Medicine Group at Zuse Institute Berlin. His main research interests are optimal control with PDEs and biomedical applications.