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COAP 2006 BEST PAPER AWARD

A local relaxation approach for the siting of electrical substation

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In each year, the Computational Optimization and Applications (COAP) editorial board selects a paper from the preceding year's COAP publications for the "Best Paper Award." The recipients of the award for papers published in 2006 are Walter Murray of Stanford University and Uday V. Shanbhag of the University of Illinois at Urbana-Champaign for their paper "A Local Relaxation Approach for the Siting of Electrical Substations," published in volume 33, pages 7–49.

This paper presents work done when Shanbhag was a doctoral student at the Systems Optimization Laboratory at Stanford University. The effort began after a former student Mukund Thapa mentioned that Stephen Chapel at the Electric Power Research Institute (EPRI) had been approached for help by Robert (Bob) Fletcher of the Snohomish Public Utility (SPU) to solve a problem of placing substations in an electrical network. Bob had apparently approached a number of companies with little success since the general response had been that the problem looked intractable. Murray, with his doctoral student Kien-Ming Ng, had been working on another discrete problem from Ericsson to assign frequencies in a cellular network and had a lot of success [5].

After an initial survey funded by SPU through Stephen Chapel at EPRI, a full-scale research effort began in 2002 with SPU and with Bob Fletcher serving as our principal utility contact. The project's core problem lay in developing efficient optimization

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methods for the location of electrical substations on rectangular grids, with the goal of minimizing installation and operating costs (from transmission losses). It may be thought that rectangular grids are a poor approximation to the actual grid. However, the nodes on the grid represent the load that may be drawn in a particular $1/2$ mile by $1/2$ mile block. The utility has a requirement to meet such a load.

It was not immediately apparent how best to pose the problem mathematically. From previous work on optimal power flow (OPF) under contingencies, it was apparent that any change to the circuit such as moving a substation from one node to another required a different set of powerflow equations. Indeed even the set of unknowns changes. In the OPF, the various configurations of the distribution in the set of constraints were simply duplicated. Typically about 10 contingencies would be accounted for. To do that here would have required an astronomically large number of constraints. Fortunately, the authors came up with a compact representation of the problem, which resulted in the need to solve a mixed-integer quadratic program (MIQP). The downside was that the number of integer and real variables could be 5000 of each. There did not appear to be any off-the-shelf software at that time that would come close to solving such a problem. At the time CPLEX could not solve discrete quadratic programs, but even later when such software was available, the largest problem which could be solved with CPLEX was about 100 variables.

In designing a new algorithm, the authors concluded that given a configuration of substations it may be necessary to move all substations simultaneously to get a better configuration. It certainly looked like such an approach was necessary to obtain the efficiency needed to solve large problems, the reasoning being, that the number of such moves would, by and large, be independent of the size of the problem if the "density" of substations was fairly constant.

The authors adopted an approach in the spirit of linesearch methods for solving smooth problems in continuous variables. Starting from an initial configuration, they determined a direction to a neighboring configuration. That is, a configuration obtained by moving substations to adjacent nodes. The difficulty was how to find this "good" neighbor. Even under the restriction of not allowing a substation to move anywhere, the number of possible moves is astronomically large. In addition a hierarchy of "good" neighbors had to be constructed to emulate the linesearch aspect of linesearch methods (or the adjustment of the trust-region in trust-region methods) that enabled an improved approximation to be determined when the initial step fails. A characteristic of this feature is that the step (or trust-region) is becoming more conservative. Since the problem is discrete, this hierarchy is finite. A node and it neighbors were grouped into 9-node stencils. These are the 9 possible locations for a given substation to be at the next iteration. To determine the "good" neighbor nodes, they first relax this local approximation and solve the resulting continuous problem, which turns out to be a large-scale convex quadratic program. This problem was solved using the active-set quadratic programming solver `SQOPT` [1]. Figure 1 provides a graphical illustration of the process.

Implicit in the aforementioned description is the availability of an initial integer feasible estimate of the solution. Obtaining such an estimate is generally quite difficult. Fortunately, the authors did not need the quality of the initial approximation to be that good (say within 30%). The approach adopted was to relax the binary variables and modify the objective function then solve the resulting continuous problem.

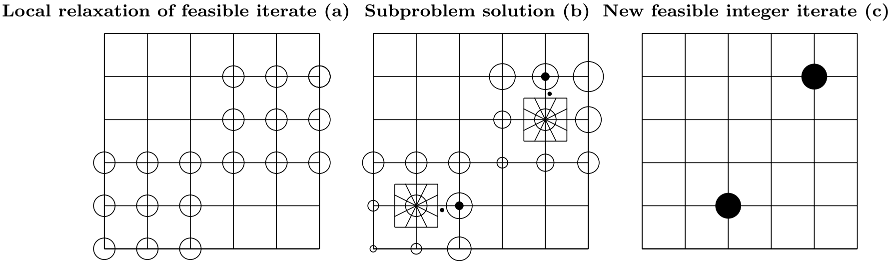

Fig. 1 Figure (**a**) shows a local relaxation around a feasible integer solution. (**b**) Shows the orthants associated with each stencil and the *small darkened circles* outside the inner stencils represent the centers-of-gravity of the stencil. The placement of the *darkened circle* in the larger circles represents a candidate position of the substations. Finally, (**c**) shows the new positions of the substations

Through a rounding procedure, it was possible to obtain a reasonable integer solution. It is observed that the SSO algorithm produces comparable optimal costs to those produced by commercial solvers such as DICOPT, SBB, BARON and CPLEX [3, 6, 7], in the few cases where comparisons were possible. Furthermore, the algorithm scales well with grid size as compared with its commercial counterparts. To exemplify the differences in time taken, the SSO algorithm takes less than a minute to solve the 15×15 case while CPLEX takes over 10 hours (on a SunSparc running Solaris 8 with 1 GB of RAM). Note that CPLEX took 3.47 CPU seconds to solve the 6×6 problem. Lower bounds for the solution are also provided by solving a sequence of mixed-integer linear programs. Results are provided for a variety of uniform and Gaussian load distributions as well as real examples from SPU. The algorithm shows slow growth in computational effort with the number of integer variables.

In addition the impact on the solution from starting at very different initial configurations was investigated. In the paper the authors illustrate the impact by showing the performance on a realistic load distribution given by the SPU on a 24×46 grid. By starting from both good and poor initial positions, it can be seen that the substations drift over significant distances to overcome a poor initial placement of substations. Since the final cost differs only by 1%, this suggests that the approach is not very sensitive to starting points. The method may be extended to general nonlinear cost functions by using a general NLP solver (such as SNOPT) [2] instead of a quadratic programming solver. Alternately, a local quadratic approximation of the nonlinear problem could be employed. In addition, the problem as posed may be embellished in a number of ways ranging from varying substation sizes to upgrading transmission lines. More generally, the authors discuss the solution of nonlinear facility location problems using similar ideas [4].

One interesting outcome of the research was the character of the solution. It can be the case when cost is minimized, the likelihood of a catastrophe is being maximized. This is of particular significance for electric utilities. Indeed although not mentioned, it is important for the system to survive a substation failure. *Survive*, in this context, implies that the voltage drop at a node should not cross a threshold. Obviously if at the optimum solution, some voltages are close to this threshold the system is in danger. It turned out the optimum solution almost maximizes the voltage gap to the threshold.

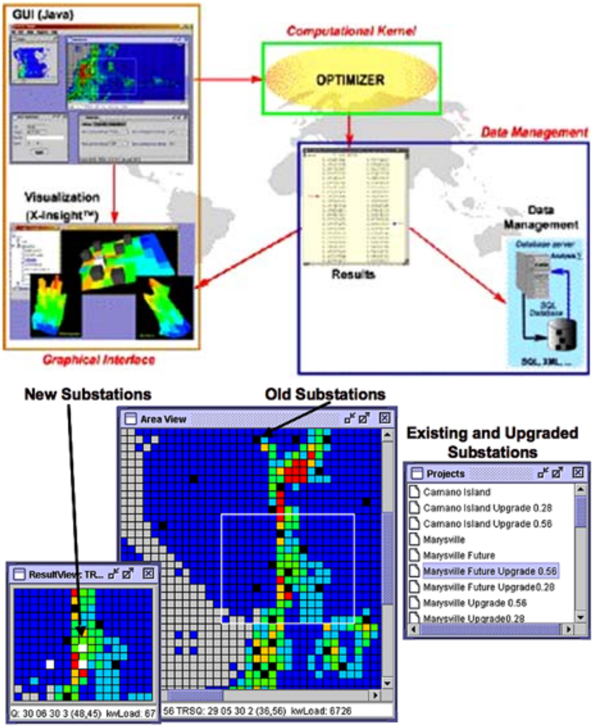

Fig. 2 Software architecture (*L*) and screenshot of results with new, upgraded and existing substations over regular grid (*R*)

Finally, the goal of the research lay in implementing the algorithm within a decision-support tool with a graphical front-end. The user interface was developed at Bergen Software Services International[1] in an effort by Dr. Patrick Gaffney and his co-workers, a Java-based front-end was developed with the capabilities of managing data, housing the computational kernel and providing a graphical interface (see Fig. 2). This product was subsequently tested and implemented at Snohomish Public Utility where it currently aids planners.

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Uday V. Shanbhag has a Ph.D. from Stanford University's department of Management Science and Engineering (February 2006), with a specialization in operations research. He also holds a masters degrees from MIT, Cambridge and an undergraduate degree from IIT, Bombay. Since 2006, he has been at the department of Industrial and Enterprise Systems Engineering at the University of Illinois at Urbana-Champaign. His dissertation (unrelated to this work) was awarded the A.W. Tucker Prize by the Mathematical Programming Society (MPS) in 2006 and he is also an NCSA Faculty Fellow (2006/07).