On-line appendix to the paper "Numerical comparison of merit function with filter criterion in Inexact Restoration algoritms using Hard-Spheres Problems"*

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Tables 1–5 list the 40 problems, displaying in their columns the number p, the number of variables and constraints, the minimum, maximum and average amounts of the final objective function, the average of the CPU time, the number of outer iterations, the number of inner iterations and elements in the filters. Note that for $n = 6$ we listed also the minimum and maximum values of these quantities. The items are listed for each of the five solvers, namely, FILTER A, FILTER B, FILTER S, MERIT and LANCELOT, respectively. The number of outer and inner iterations are listed just for the four inexact restoration algorithms.

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Problem size			Final objective function			Time	Outer	Inner	Filter
p	var.	cons.	min.	max.	ave.	sec.	it.	it.	pairs
8	25	36	1.2155625	1.2155625	1.2155625	0.03	21.6	51.7	4.3
			1.2155625	1.2155625	1.2155625	0.03	21.6	51.9	21.2
			1.2155625	1.2155625	1.2155625	0.03	21.3	51.5	3.8
			1.2155625	1.2155625	1.2155625	0.03	21.4	51.4	–
			1.2155625	1.2155625	1.2155625	0.03	–	–	–
9	28	45	1.1547005	1.1547005	1.1547005	0.03	20.9	50.3	3.0
			1.1547005	1.1547005	1.1547005	0.03	20.9	50.3	20.6
			1.1547034	1.1547034	1.1547034	0.03	20.7	50.2	2.6
			1.1547005	1.1547005	1.1547005	0.03	21.5	50.7	–
			1.1547034	1.1547034	1.1547034	0.04	–	–	–
10	31	55	1.0514622	1.0914263	1.0870816	0.04	21.1	50.8	3.5
			1.0514622	1.0914263	1.0870816	0.04	21.1	50.8	20.9
			1.0514622	1.0914263	1.0870995	0.04	21.1	50.9	3.2
			1.0514622	1.0914263	1.0878799	0.04	21.2	50.6	–
			1.0514656	1.0914303	1.0901675	0.05	–	–	–
11	34	66	1.0514622	1.0514622	1.0514622	0.05	20.5	49.8	3.6
			1.0514622	1.0514622	1.0514622	0.05	20.5	49.8	20.4
			1.0514622	1.0514622	1.0514622	0.05	20.5	49.8	3.3
			1.0514622	1.0514622	1.0514622	0.05	20.3	50.4	–
			1.0514656	1.0514656	1.0514656	0.06	–	–	–
12	37	78	0.9472196	1.0514622	1.0493774	0.07	20.7	49.9	3.5
			0.9472196	1.0514622	1.0493774	0.06	20.7	49.9	20.6
			0.9472196	1.0514622	1.0493774	0.07	20.7	50.0	3.4
			0.9472196	1.0514622	1.0493774	0.07	21.0	51.6	–
			0.9463826	1.0514766	1.0388557	0.07	–	–	–
13	40	91	0.9281797	0.9535789	0.9484569	0.07	20.8	50.5	3.2
			0.9281797	0.9535789	0.9484569	0.07	20.6	50.2	20.5
			0.9281797	0.9535789	0.9484569	0.07	20.5	50.0	2.7
			0.9281797	0.9535789	0.9488610	0.07	20.4	50.2	–
			0.9281810	0.9564099	0.9496905	0.12	–	–	–
14	43	105	0.9091201	0.9338626	0.9287286	0.08	21.2	51.1	2.8
			0.9091201	0.9338626	0.9282729	0.09	21.2	51.2	21.0
			0.9091201	0.9338626	0.9284086	0.08	20.1	50.7	2.3
			0.9091201	0.9338626	0.9285222	0.08	21.6	52.3	–
			0.9025187	0.9338629	0.9297187	0.15	–	–	–
15	46	120	0.8745439	0.9026562	0.9006571	0.10	21.6	51.9	3.3
			0.8745439	0.9026562	0.9006571	0.10	21.6	51.9	21.4
			0.8745439	0.9026562	0.9006571	0.10	21.4	51.8	2.6
			0.8745439	0.9026562	0.9006715	0.09	21.4	51.2	–
			0.8978196	0.9026517	0.9017889	0.20	–	–	–
16	49	136	0.8650118	0.8805741	0.8747759	0.11	21.3	51.2	3.2
			0.8650118	0.8805741	0.8747759	0.11	21.1	51.0	20.1
			0.8650118	0.8805741	0.8747759	0.11	20.7	50.5	2.6
			0.8650118	0.8805741	0.8748141	0.10	21.7	51.5	–
			0.8650087	0.8805794	0.8772048	0.24	–	–	–
17	52	153	0.8426190	0.8624449	0.8608943	0.13	21.5	51.5	3.2
			0.8426190	0.8624449	0.8608943	0.12	21.4	51.5	21.0
			0.8426190	0.8624449	0.8608943	0.12	21.2	51.4	2.5
			0.8426190	0.8624449	0.8608943	0.12	21.4	51.4	–
			0.8399286	0.8624500	0.8600915	0.28	–	–	–

Table 1: Final objective function, CPU times, number of outer iterations, inner iterations and elements added in the filter for $n = 3$ using Filter A, Filter B, Filter S, MERIT and LANCELOT algorithms.

Problem size			Final objective function			Time	Outer	Inner	Filter
p	var.	cons.	min.	max.	ave.	sec.	it.	it.	pairs
20	81	210	1.0351487	1.0637119	1.0612852	0.36	22.8	53.7	3.3
			1.0351487	1.0637119	1.0610858	0.35	22.5	53.3	21.3
			1.0351487	1.0637119	1.0608863	0.35	22.1	53.3	2.9
			1.0351487	1.0637119	1.0608446	0.33	21.2	52.0	–
			1.0351232	1.0637105	1.0607658	0.53	–	–	–
21	85	231	0.9994649	1.0282209	1.0240093	0.36	24.0	55.0	3.5
			0.9994649	1.0282209	1.0240159	0.36	24.0	55.1	20.3
			0.9994649	1.0282209	1.0238791	0.37	23.6	55.4	3.0
			0.9994649	1.0282209	1.0235018	0.34	22.6	54.2	–
			0.9916754	1.0282218	1.0230901	1.08	–	–	–
22	89	253	0.9833497	1.0019895	0.9961807	0.41	23.3	54.0	3.5
			0.9833497	1.0019895	0.9961904	0.41	23.3	54.3	20.1
			0.9833497	1.0019895	0.9963420	0.42	22.7	53.9	2.7
			0.9781643	1.0019895	0.9960382	0.42	22.1	55.2	–
			0.9825274	1.0011988	0.9956717	1.20	–	–	–
23	93	276	0.9729942	1.0000000	0.9831889	0.49	22.9	53.7	3.6
			0.9729942	1.0000000	0.9830452	0.49	22.6	53.4	20.2
			0.9729942	1.0000000	0.9830373	0.51	21.9	53.0	3.0
			0.9729942	1.0000000	0.9830550	0.47	22.0	53.6	–
			0.9748538	0.9918568	0.9840580	1.73	–	–	–
24	97	300	0.9583287	1.0000000	0.9756047	0.54	25.1	56.6	3.7
			0.9583287	1.0000000	0.9755996	0.55	25.0	56.6	19.9
			0.9583287	1.0000000	0.9757404	0.56	25.5	58.5	3.0
			0.9583287	1.0000000	0.9755052	0.57	25.8	59.3	–
			0.9599271	0.9828733	0.9746482	1.94	–	–	–
25	101	325	0.9494635	0.9619544	0.9573685	0.62	24.5	55.8	3.9
			0.9494635	0.9619544	0.9573017	0.63	24.4	55.9	19.8
			0.9494635	0.9619544	0.9573019	0.63	24.2	56.3	3.0
			0.9494635	0.9619430	0.9574099	0.61	24.0	58.2	–
			0.9470903	0.9619563	0.9575385	2.12	–	–	–
26	105	351	0.9364391	0.9583427	0.9478305	0.73	24.4	55.9	4.2
			0.9364391	0.9583427	0.9478037	0.72	24.6	56.6	20.3
			0.9364391	0.9583427	0.9478143	0.70	24.7	57.7	3.7
			0.9364391	0.9583427	0.9479445	0.70	25.2	59.0	–
			0.9348048	0.9583423	0.9507970	2.46	–	–	–
27	109	378	0.9231654	0.9390756	0.9347026	0.81	24.0	55.5	4.0
			0.9231654	0.9389281	0.9348128	0.81	23.9	55.5	19.4
			0.9231654	0.9389281	0.9348574	0.83	23.8	56.2	3.3
			0.9231654	0.9389281	0.9346958	0.78	23.6	56.5	–
			0.9257105	0.9400213	0.9347336	3.25	–	–	–
28	113	406	0.9166071	0.9301962	0.9241076	0.93	24.1	55.7	3.9
			0.9166071	0.9301962	0.9240767	0.94	24.1	55.8	19.9
			0.9166071	0.9301962	0.9239558	0.91	23.3	55.1	3.4
			0.9166071	0.9301962	0.9241882	0.86	24.7	57.1	–
			0.9175620	0.9301398	0.9252961	3.98	–	–	–
29	117	435	0.9069596	0.9210020	0.9156587	1.05	25.3	57.1	4.2
			0.9069596	0.9194775	0.9155891	1.04	25.2	57.1	19.6
			0.9069596	0.9198191	0.9157508	1.07	25.7	58.6	3.44
			0.9069596	0.9198191	0.9155294	1.03	25.0	59.2	–
			0.9090765	0.9211515	0.9154703	4.31	–	–	–

Table 2: Final objective function, CPU times, number of outer iterations, inner iterations and elements added in the filter for $n = 4$ using Filter A, Filter B, Filter S, MERIT and LANCELOT algorithms.

Problem size			Final objective function			Time	Outer	Inner	Filter
p	var.	cons.	min.	max.	ave.	sec.	it.	it.	pairs
35	176	630	1.0019034	1.0173904	1.0110672	3.52	28.4	60.6	2.7
			1.0019034	1.0173904	1.0111020	3.32	28.7	60.8	16.3
			1.0019034	1.0173881	1.0109556	3.92	29.1	63.6	2.1
			1.0028215	1.0173952	1.0114531	4.29	28.9	68.1	–
			1.0020579	1.0185971	1.0117657	16.92	–	–	–
36	181	666	0.9987403	1.0135038	1.0057361	4.74	36.1	69.8	2.8
			0.9987403	1.0137977	1.0057360	4.36	35.8	69.6	16.7
			0.9987403	1.0135038	1.0058082	4.87	35.8	70.9	2.2
			0.9987403	1.0135820	1.0056463	5.45	33.0	76.2	–
			0.9956405	1.0130844	1.0052108	19.43	–	–	–
37	186	703	0.9904754	1.0040832	0.9978468	4.91	31.8	65.3	2.9
			0.9904754	1.0040832	0.9978910	4.79	32.5	66.0	16.6
			0.9904754	1.0040832	0.9978472	5.65	35.9	72.2	2.2
			0.9907692	1.0033062	0.9979096	5.73	31.4	73.6	–
			0.9931365	1.0041613	0.9983200	21.65	–	–	–
38	191	741	0.9852629	0.9989065	0.9925839	5.95	34.9	68.6	2.9
			0.9852629	0.9989091	0.9928450	5.91	35.8	69.9	17.2
			0.9852629	0.9989091	0.9929870	6.64	39.0	76.2	2.2
			0.9851205	1.0004915	0.9927512	7.20	35.0	79.5	–
			0.9848553	0.9988393	0.9926246	26.96	–	–	–
39	196	780	0.9794599	0.9928665	0.9868229	6.33	34.7	69.5	3.1
			0.9794710	0.9928664	0.9867493	5.70	34.1	68.5	17.0
			0.9794710	0.9957697	0.9866839	7.05	35.6	74.0	2.4
			0.9793344	0.9928664	0.9864580	7.97	34.1	79.2	–
			0.9790812	0.9949472	0.9874984	27.94	–	–	–
40	201	820	0.9732567	0.9869823	0.9806507	6.64	32.1	65.7	3.3
			0.9732567	0.9869823	0.9806920	6.39	33.3	67.1	16.8
			0.9746794	0.9923988	0.9814932	8.11	38.3	75.4	2.5
			0.9732567	0.9923987	0.9811260	9.70	38.4	85.1	–
			0.9757356	0.9881397	0.9816043	29.78	–	–	–
41	206	861	0.9672959	0.9828964	0.9755498	7.70	35.0	69.3	3.0
			0.9672959	0.9828965	0.9754632	6.99	32.8	66.8	17.0
			0.9681821	0.9827456	0.9757563	9.54	36.5	74.4	2.7
			0.9681821	0.9830889	0.9752849	9.29	34.0	77.2	–
			0.9671918	0.9833616	0.9756260	39.22	–	–	–
42	211	946	0.9557084	0.9764238	0.9682565	8.67	33.0	67.2	2.9
			0.9557084	0.9764238	0.9686490	8.78	34.1	68.6	16.2
			0.9557037	0.9776676	0.9683376	10.44	38.4	76.3	2.5
			0.9559526	0.9768630	0.9679390	11.02	38.0	86.4	–
			0.9618628	0.9758893	0.9700121	45.67	–	–	–
43	216	946	0.9548309	0.9702342	0.9628405	9.78	35.4	69.3	3.0
			0.9548310	0.9702343	0.9628573	9.59	34.6	68.3	15.8
			0.9568459	0.9701901	0.9633174	12.42	42.4	80.1	2.2
			0.9561244	0.9716775	0.9631215	12.35	40.2	89.8	–
			0.9578622	0.9715246	0.9632089	49.20	–	–	–
44	221	990	0.9507123	0.9626061	0.9566949	9.73	31.0	64.4	3.0
			0.9507124	0.9626061	0.9567021	9.88	30.7	64.1	16.0
			0.9507124	0.9633598	0.9568399	11.80	35.2	72.5	2.5
			0.9501656	0.9619014	0.9566083	13.83	37.2	83.6	–
			0.9529848	0.9634833	0.9580063	58.75	–	–	–

Table 3: Final objective function, CPU times, number of outer iterations, inner iterations and elements added in the filter for $n = 5$ using Filter A, Filter B, Filter S, MERIT and LANCELOT algorithms.

Problem size			Final objective function			CPU time (seconds)		
p	var.	constr.	min.	max.	average	min.	max.	average
65	391	2145	0.9750307	0.9839771	0.9788601	76.23	1906.09	210.13
			0.9750307	0.9839771	0.9788941	86.27	2138.07	215.23
			0.9746794	0.9834679	0.9791341	93.04	867.58	193.48
			0.9749358	0.9835330	0.9796404	98.02	1254.27	280.70
			0.9752128	0.9829446	0.9797405	111.82	679.81	378.62
66	397	2211	0.9702609	0.9794957	0.9757566	90.30	420.27	189.15
			0.9702609	0.9794957	0.9758951	98.81	660.63	217.69
			0.9704975	0.9793902	0.9759570	117.51	1290.53	294.85
			0.9696282	0.9801942	0.9758308	103.11	1364.28	291.10
			0.9717201	0.9800714	0.9763078	137.84	1398.50	387.52
67	403	2278	0.9687860	0.9761731	0.9734107	107.72	1111.14	202.71
			0.9687860	0.9777242	0.9736186	121.02	1249.80	249.83
			0.9696197	0.9772794	0.9739346	124.50	1222.24	296.99
			0.9679853	0.9768840	0.9734019	111.43	789.50	312.48
			0.9688860	0.9774866	0.9737456	143.50	939.09	412.53
68	409	2346	0.9659695	0.9747148	0.9703198	103.67	1431.63	228.18
			0.9659695	0.9747148	0.9703313	104.89	1548.73	251.60
			0.9656169	0.9741291	0.9705158	94.19	1016.96	278.09
			0.9660792	0.9739831	0.9704795	116.10	708.86	284.80
			0.9670057	0.9753973	0.9709359	165.93	926.63	503.76
69	415	2415	0.9601896	0.9705048	0.9671616	85.60	2116.95	283.55
			0.9601896	0.9705048	0.9671695	92.64	2251.17	301.61
			0.9646882	0.9709664	0.9674271	151.08	710.69	308.73
			0.9610354	0.9724445	0.9678093	134.98	1096.48	398.41
			0.9635144	0.9724094	0.9684886	182.21	1551.04	563.82
70	421	2485	0.9599638	0.9682884	0.9651296	105.08	567.04	240.11
			0.9599638	0.9682884	0.9650872	113.65	437.56	248.55
			0.9595804	0.9682882	0.9649263	172.65	3800.57	381.99
			0.9611661	0.9687052	0.9651688	161.59	1155.63	390.09
			0.9609474	0.9685350	0.9650469	173.81	1463.32	508.44
71	427	2556	0.9575086	0.9666411	0.9621436	151.52	1081.69	322.27
			0.9575086	0.9645867	0.9620446	152.18	1123.82	304.86
			0.9584651	0.9653407	0.9622515	170.95	2421.21	465.10
			0.9584438	0.9650520	0.9621290	164.39	1212.50	527.13
			0.9565459	0.9665299	0.9625105	173.10	1695.82	636.79
72	433	2628	0.9536736	0.9632995	0.9590842	148.64	3273.91	372.16
			0.9536736	0.9632995	0.9589869	149.44	3289.58	365.31
			0.9557052	0.9629132	0.9597783	162.78	1446.89	395.58
			0.9562493	0.9634698	0.9595599	166.95	1503.07	513.85
			0.9542012	0.9644688	0.9597164	210.90	1974.04	716.89
73	439	2701	0.9532062	0.9608750	0.9571851	179.19	2243.47	381.25
			0.9532063	0.9608750	0.9571972	171.44	2219.15	384.14
			0.9528294	0.9604783	0.9571442	154.76	3269.99	497.94
			0.9509735	0.9594646	0.9566721	187.23	1601.61	545.68
			0.9533205	0.9619356	0.9572629	184.02	2089.80	707.54
74	445	2775	0.9510839	0.9601798	0.9547268	172.69	699.36	342.33
			0.9510839	0.9601798	0.9546246	173.01	751.06	344.54
			0.9500169	0.9583462	0.9548356	163.38	5480.95	645.48
			0.9503846	0.9584177	0.9545539	234.81	2051.10	663.39
			0.9489152	0.9578204	0.9545271	264.20	2630.08	853.60

Table 4: Final objective function and CPU times for $n = 6$ using Filter A, Filter B, Filter S, MERIT and LANCELOT algorithms.

Problem size			Outer iterations			Inner iterations			Elements in F		
p	var.	constr.	min.	max.	ave.	min.	max.	ave.	min.	max.	ave.
65	391	2145	23	701	76.5	55	740	115.8	2	3	3.2
			23	701	71.0	55	740	110.4	10	18	13.9
			23	279	63.4	57	325	110.3	1	5	2.2
			23	269	74.2	57	548	159.4	–	–	–
66	397	2211	26	357	63.6	63	400	103.1	1	5	3.3
			26	214	68.2	56	253	107.4	8	19	13.0
			27	421	93.1	64	454	141.4	1	5	2.4
			26	297	72.7	61	604	155.1	–	–	–
67	403	2278	29	357	63.6	63	400	103.1	1	5	3.3
			29	357	69.6	63	400	109.5	10	22	14.2
			32	382	84.4	69	443	137.6	1	5	2.4
			25	170	71.2	59	351	152.6	–	–	–
68	409	2346	29	374	63.5	62	423	102.3	2	5	3.2
			28	374	65.5	62	423	104.4	8	21	13.42
			29	292	76.7	64	348	125.4	1	4	2.4
			26	123	62.6	61	256	134.3	–	–	–
69	415	2415	22	513	73.7	53	557	113.2	2	5	3.4
			22	513	73.1	53	557	112.7	9	20	13.5
			28	188	74.2	67	234	122.6	2	4	2.6
			26	215	78.5	61	439	166.7	–	–	–
70	421	2485	30	157	61.6	65	196	102.2	1	5	3.1
			32	118	59.2	65	167	99.4	7	20	12.8
			29	802	82.6	62	866	133.8	1	5	2.3
			32	198	74.4	74	407	159.1	–	–	–
71	427	2556	28	274	74.2	62	314	114.3	1	5	3.6
			28	274	68.1	62	314	108.4	9	20	12.7
			31	505	95.0	70	556	147.6	1	4	2.3
			28	164	77.3	66	339	164.0	–	–	–
72	433	2628	26	730	84.6	61	766	125.1	2	6	3.9
			26	730	82.2	61	766	122.4	9	23	13.2
			27	329	82.1	64	373	134.0	1	4	2.3
			28	195	81.6	66	401	172.2	–	–	–
73	439	2701	33	448	81.9	69	489	122.5	1	5	3.0
			33	448	81.7	69	489	122.4	8	19	12.5
			29	652	98.2	68	708	150.5	1	5	2.4
			29	221	79.1	68	451	169.9	–	–	–
74	445	2775	31	151	70.2	68	190	111.8	1	6	3.6
			31	171	69.3	68	218	110.7	8	19	12.7
			25	1000	121.1	60	1046	174.0	1	4	2.4
			33	242	87.3	76	496	185.0	–	–	–

Table 5: Outer iterations, Inner iterations and elements in the filter for $n = 6$, using Filter A, Filter B, Filter S and MERIT algorithms.